

# PSAMMODYNAMICS: ATTRACTORS AND ENERGETICS

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**Abstract.** *Psammodynamics denotes a combination of seismo- and thermodynamics for sand-like materials. Monotonous deformations of samples lead first to an alignment of the hidden state and then to a state limit. These two attractors are partly reproduced by elastoplastic or hypoplastic relations, and also with an energetic approach, this enables validations and calibrations. Monotonous plus cyclic deformations of samples lead to asymptotic stress cycles, these attractors are partly reproduced by elastoplastic or hypoplastic relations with hidden variables and by the energetic approach. The average evolution up to attractors is captured by a novel seismo-hypoplastic relation with energetic support. Stabilization and destabilization with pulsating seismicity are introduced with tower models, they mean an attractor or a repeller. A collapse at the verge of convexity of the free energy has a mode which is related with an eigenvector. This concept could be extended for solids and thermal activation.*

## 1 INTRODUCTION

*Attractors* were observed in reality and enhanced physical theories. A group of galaxies behind Virgo tends to a black hole, this Great Attractor corresponds to a monotonous attractor (asymptote) of the equations of motion and tends to deterministic chaos indicating a strange attractor. More often cosmic clouds tend to state cycles, e.g. the Solar system, but can also get chaotic. Newton explained planetary cycles, Poincaré introduced for it the chaos theory. Closed aggregates of atoms and molecules with attraction and repulsion tend to thermodynamic equilibria or to reaction cycles, and get chaotic at critical points. Atoms have periodic quantum states which result as eigenmodes of Schrödinger's equation.

Such systems are *conservative*, i.e. particles are not changed by interactions or are exactly reconstituted in reactions, therefore mutual forces are given by relative positions via gradients of potential energies. This enables permanent state cycles at different scales, linear evolution equations and Boltzmann statistics. A stable compound of sand, water and solids remains at equilibrium if it is not disturbed. Otherwise rearrangements and force redistributions change the grains at least a little, so interaction forces are not determined by potentials and relative positions, energy is dissipated and no two grains are equal. This means that granular systems are *not conservative*. Monotonous, cyclic and strange attractors can only occur therefore by driving with energy supply, and the granular degradation should be taken into account.

Cylindrical *psammoid* (i.e. sand-like) samples have two degrees of freedom, axial and radial, if they are uniform. Their state is described by two Cauchy stress components ( $\sigma_1$  and  $\sigma_2 = \sigma_3$ ), the void ratio  $e$  and internal variables which are hidden (Physical Soil Mechanics, Gudehus 2010, PSM in the sequel). As outlined in Sec. 2 proportional deformations lead first to an alignment of the hidden state so that this is no more needed explicitly. Continued deformation leads to a state limit where the sample loses its uniformity by localized shearing with dilation or contraction. These two attractors are partly reproduced by elastoplastic and hypoplastic relations without hidden variables, this serves to delimit the range of validity and to calibrate parameters therein (PSM).

The partial success of this approach calls for a physical explanation by means of *energies*. This was qualitatively achieved with a theory named Granular Solid Hydrodynamics (GSH, Jiang and Liu 2009). In the

forthcoming paper 'Seismo- and thermodynamics of granular solids' (Gudehus et al. 2010, STG in the sequel) this theory was supported by further arguments, modified and quantified. Instead of GSH it may be called *psammodynamics* as psammoids are matter in its own right. This concept is shown in Sec. 2 to reproduce hypoplastic relations within bounds given by the two attractors mentioned above.

Attractors for non-monotonous evolutions of cylindrical samples are introduced in Sec. 3. As long as localization and decay are avoided combinations of isochoric monotonous and cyclic deformations lead to stress cycles, these attractors serve to judge and quantify elastoplastic and hypoplastic relations with hidden variables (PSM). A unified representation enables an evolution equation relating average changes of shape and state. This rate-independent equation is supported by STG, but more experiments and simulations will be needed for a detailed validation and calibration.

Systems with sand and attached solids are considered in Sec. 4. Model towers serve to introduce the total free energy which is needed to judge stability, stabilization and destabilization. Other than with an elastic ground sand gets rearranged by pulsating seismicity, with it the system can have an attractor or a repeller. The novel constitutive model is thought to predict gradual changes of position and state, whereas an impending collapse can as yet at best be estimated. Pore water and elastic structures can be incorporated, this concept works for a wide range of initial and boundary conditions. After a conclusion it is indicated in Sec. 4 how it could be applied for solids and with thermal activation.

## 2 MONOTONOUS DEFORMATIONS OF SAMPLES

Some evolutions of cylindrical sand samples (like in perfect so-called triaxial tests, PSM) are shown in Fig. 1. The imposed strain paths (a, axial  $\varepsilon_1$ , radial  $\varepsilon_2 = \varepsilon_3$ ) may be proportional with contraction (A), constant volume (B) or dilation (C). The obtained stress path (b) tends to a straight line from the origin (A), a point (B) or a straight line towards the origin (C), respectively. This behavior is also visible from *response polars* which represent stress rates for unit strain rates with different directions. As plotted they resemble ellipses which are bigger for higher pressures and more eccentric for bigger ratios of shear and normal stress. A plot of void ratio  $e$  versus mean pressure  $p = (\sigma_1 + 2\sigma_3)/3$  exhibits an approach to limit curves (c) with  $p$ -increase (A), stationary  $p$  (B) or  $p$ -decrease (C).

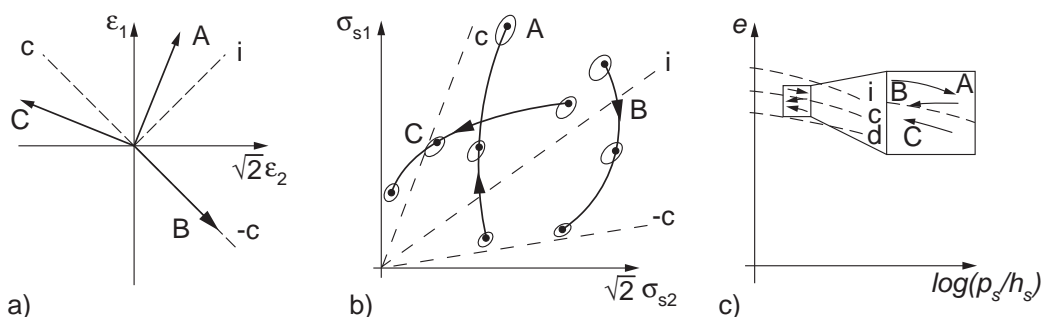


Figure 1: a) Proportional deformations of a cylindrical psammoid sample, b) induced stress paths with response polars, c) induced evolutions of void ratio versus pressure

These evolutions, and similar ones with monotonous strain paths, are characterized by two kinds of attractors. First the hidden internal state is *aligned* by the deformation to the stress state so that it is not needed explicitly in constitutive relations. Thereafter the sample approaches a *state limit* with a loss of uniformity, viz. compaction bands (A), shear bands (B) or decay (C). Elastoplastic relations with density hardening capture contractant and isochoric paths except for very low or high pressures, very dilatant paths with low relative void ratios are missed. Hypoplastic relations imply more realistic response polars (Kolymbas and Wu 1993) and work for a wider range of void ratios and pressures. Both models ignore the spontaneous loss of uniformity near state limits. Shear localizations can be captured by means of additional polar quantities, they arise spontaneously with bifurcation (Vardoulakis and Georgopoulos 2005) and can tend to fractal patterns (PSM).

The grains are jammed so that they have a specific *elastic energy*  $w_e$  which is a function of the average elastic strain components  $u_i$  and the density  $\rho$  or void ratio  $e$ . The elastic stress  $\pi_i$  is conjugated with  $u_i$  via  $w_e$  by  $\pi_i = -\partial w_e / \partial u_i$ . Due to increasing contact flats  $w_e$  should depend on  $\Delta \equiv -\Sigma u_i$  by  $w_e \propto \Delta^m$  with an exponent

$m$  according to the grain roughness. Jiang and Liu (2009) propose  $m = 2.5$ , but  $m = 3$  would suit better to the observed stiffness  $\partial p/\partial \Delta \propto p^{1/2}$ . At the verge of convexity of  $w_e$  the grain skeleton is no more stable, we will see below what that means. Jiang and Liu use a critical friction ratio for this verge, and an elaborate dependence on  $\rho$  so that  $w_e \propto \Delta^{2.5}$  is slightly violated.

The crackling noise during deformations indicates a heat-like *seismic energy*  $w_s$  which is not stored. In GSH this depends on a granular or seismic temperature  $T_g$  by  $w_s = b\rho T_g^2/2$ . The grains get totally unjammed by minute continuous shaking so that  $w_e$  vanishes and the pressure gets purely seismic and hydrostatic. Such seismodynamic equilibria with high  $\rho$  are stable due to  $w_s \propto T_g^2$ . Strong shaking causes granular boiling with expansion and convection cells, then  $w_s \propto T_g$  holds as for a granular gas. Experiments indicate a boiling temperature  $T_b \approx 10^9$  K like in a Red Giant. Thus the factor  $b$  was estimated, this leads to  $w_s < 10^{-5}w_e$  for deformations with rates  $D < 10^{-3}/s$  and  $p > 1\text{kPa}$  (STG).

The seismic energy gets lost into heat with a rate  $\dot{w}_s \propto -w_s$  and a half-life  $t_s < \approx 10^{-3}s$  for  $D > 10^{-5}/s$  as kinks lose their energy after passing a certain number of grain contacts.  $w_s$  is generated by rearrangements (intensity  $D = \sqrt{\epsilon_1^2 + 2\epsilon_2^2}$ ) with a granular viscosity  $\eta_g$ . The balance of seismic energy by GSH reads

$$\frac{b\rho}{2} \frac{dT_g^2}{dt} = -\gamma T_g^2 + \eta_g D^2 \quad (1)$$

wherein  $\gamma$  and  $\eta_g$  increase with  $T_g$  from low values for  $T_g = 0$ . For slow monotonous deformations  $T_g = D\sqrt{\eta_g/\gamma}$  is obtained as  $D$  is practically constant for much longer times than the seismic half-life  $t_s$ . With a factor  $\lambda$  which will be explained below  $\lambda\sqrt{\eta_g/\gamma}$  ranges from ca. 1 for  $T_g = 0$  to ca. 100 for  $T_g > T_h \approx 100\text{K}$  (STG).

The elastic strain  $u_i$  changes by stretching and seismic relaxation within GSH via

$$\dot{u}_i = (1 - \alpha)\dot{\epsilon}_i - \lambda T_g(u_i^* + \delta_i \Delta/10) \quad (2)$$

with  $u_i^*$  for the deviatoric part. The *transmission factor*  $\alpha$  depends on  $T_g$  by  $\alpha \approx \alpha_h \tanh(T_g^2/T_h^2)$  (STG) with  $\alpha_h \approx 0.8$  (GSH). The Cauchy stress  $\sigma_i$  is smaller than the elastic one  $\pi_i$  by  $\sigma = (1 - \alpha)\pi_i$  as the contacts are seismically softened. Imagine a bicycle with a belt connecting smooth wheels instead of a chain connecting indented wheels. Stick-slip by shaking reduces the transmission and the driving moment by the same factor, similarly grain chains are imperfect and soft by seismicity.

Combining these relations yields the stress rate

$$\dot{\sigma}_i = (1 - \alpha)\dot{\pi}_i - \dot{\alpha}\pi_i = -(1 - \alpha)H_{ij} \left[ (1 - \alpha)\dot{\epsilon}_j - D\lambda\sqrt{\frac{\eta_g}{\gamma}}(u_i^* - \delta_i \Delta/10) \right] - \dot{\alpha}\pi_i \quad (3)$$

with the differential stiffness  $H_{ij} = \partial\pi_i/\partial u_j$ . In case of a monotonous deformation with constant intensity  $D$   $\alpha$  tends to  $\alpha_h$ ,  $\lambda\sqrt{\eta_g/\gamma}$  to ca. 100 and  $T_g$  gets stationary. Then (3) goes over into a hypoplastic relation which is thus energetically justified (GSH). The transition to the first attractor introduced above is indicated by a stationary crackling noise, which is in fact pressure-independent and louder for higher  $D$ .

In the hypoplastic range, with  $T_b \gg T_g > T_h \approx 100\text{K}$ ,  $\eta_g \propto T_g$  and  $\gamma \propto T_g$  holds almost exactly by GSH and STG so that  $T_g$  and  $D$  are simultaneously constant. This can be attributed to a rate of seismic kinks  $\propto D$  due to evolving granular force chains with a constant transmission factor  $\alpha$ . With higher  $D$  and thus  $T_g$  seismic kinks are more often generated (bigger  $\eta_g$ ) and propagate less far due to a stronger seismically triggered dissipation (bigger  $\gamma$ ). The acoustic emission reflects a part of this seismic activity as some spectral fractions are enhanced by resonance of the sample in the apparatus, while higher fractions are damped or not audible. The *seismometry* is not as well developed as the thermometry.

Consider e.g. a sample with  $p = 100\text{kPa}$  and  $D = 10^{-3}/s$ . It has  $w_e \approx 10\text{J/m}^3$ ,  $T_g \approx 10^5\text{K}$ ,  $w_s \approx 10^{-6}\text{J/m}^3$ , and a seismic half-life  $t_s \approx 10^{-5}s$ . If a group of grains is at its verge of stability a minute seismic wave triggers a kind of snap-through. This *micro-hysteresis* produces heat at sliding contact flats (as observed by Luong 1982 with thermography) and a kink with far less energy which is propagated and dissipated as indicated above. The dissipation power by STG is  $Q \approx pD$  as required for dry friction. With  $D = 10^{-6}/s$  the seismic temperature  $T_g$ , the rate of kinks, the inverse of the half-length of kink propagation and of the seismic half-life  $t_s$ , and  $Q$  are 100 times smaller than with  $D = 10^{-3}/s$  for the same  $p$ .

With on-going deformation a *stability limit* is attained where the elastic energy  $w_e$  is no more convex. A stronger and wilder crackling noise indicates a dramatic increase of seismicity, the approach to a state limit is thus observable. The eigenvector of  $w_e$  for the verge of convexity determines a collapse mode  $\dot{\epsilon}_2/\dot{\epsilon}_1$  for which the imposed work is dissipated with maximal seismicity (STG). A similar maximality was assumed by Vardoulakis et al. (1978) for the bifurcation into localized shearing. The increasing fluctuations cause fractal patterns of shear bands (PSM) with polar quantities (Vardoulakis and Georgopoulos 2005), the emerging new degrees of freedom imply a granular chain reaction. Other than with thermodynamic systems such critical phenomena require energy supply as the granular interactions are not conservative.

### 3 PULSATING DEFORMATIONS OF SAMPLES

A water-saturated undrained sand sample may be deformed with an amplitude  $\epsilon_c$  and an increase  $\epsilon_a$  per cycle, Fig. 2a. The zig-zag stress path tends to an *asymptotic cycle* which is butterfly-like for  $\epsilon_a = 0$  and lenticular for  $\epsilon_a/\epsilon_c \geq \text{ca. } 1/100$  (b). This driven attractor attains the critical ratio of shear and normal stress twice for  $\epsilon_a = 0$  and once otherwise. With low  $\epsilon_c$ ,  $\epsilon_a = 0$  and moderate  $\rho$  the average pressure  $\bar{p}$  tends to zero, whereas an asymptotic  $\bar{p} > 0$  is obtained with sufficient  $\epsilon_a/\epsilon_c$ .

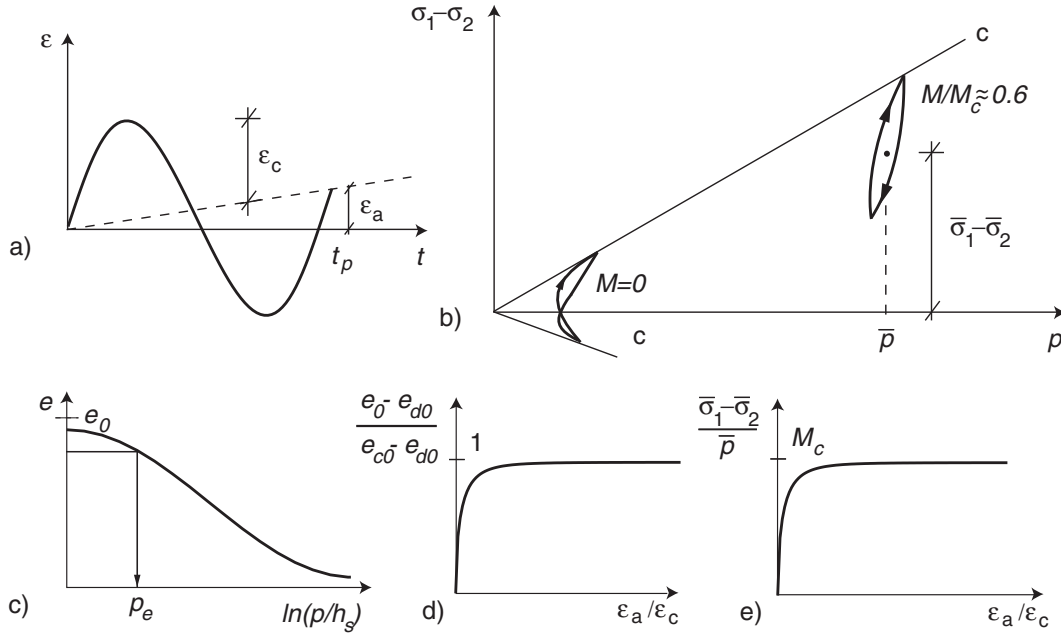


Figure 2: a) Deformation of a sample with pulsation and isochoric trend, b) asymptotic stress cycles without (left) and with isochoric trend (right), c) asymptotic void ratio versus pressure and equivalent pressure for an attractor, relative void ratio for  $p = 0$  (d) and average stress ratio (e) versus ratio of trend and amplitude for attractors

These attractors and the approach to them are well reproduced, except for small  $\epsilon_a/\epsilon_c$  and low  $\rho$ , by an elastoplastic relation with back stress (Taiebat and Dafalias 2007). The shortcoming could be removed with limit void ratios as in Fig.1c (PSM). The attractors are obtained by a hypoplastic model with intergranular strain (Niemunis and Herle 1997) for a wider range of  $\epsilon_a/\epsilon_c$  and  $\rho$ , but with flatter butterflies and faster approach than observed (PSM). Niemunis et al. (2005) proposed an *accumulation model* for the evolution of average shape and state with pulsations. It requires many empirical factors from triaxial tests and can lead close to the indicated attractors except for big  $\epsilon_a/\epsilon_c$ , then the average stress ratio  $(\bar{\sigma}_1 - \bar{\sigma}_2)/\bar{p}$  is critical so that  $(\sigma_1 - \sigma_2)/p$  is temporarily overcritical. The attractors for it can be captured by

$$\bar{p} = p_e(a_c \epsilon_c^m + M/M_c) ; \bar{\sigma}_1 - \bar{\sigma}_2 = M\bar{p} \quad (4)$$

with an  $e$ -equivalent pressure  $p_e$  by Fig. 2c and d, and a stress ratio  $M$  by Fig. 2e. The exponent is  $m \approx 2$ , the maximal  $M = M_c$  is given by a critical friction angle.  $M$  and the relative void ratio  $r_{e0} = (e_0 - e_{do})/(e_{co} - e_{do})$  increase from zero to a plateau for growing  $\epsilon_a/\epsilon_c$ .  $M_c, e_{co}, e_{do}$  and the granulate hardness  $h_s$  are hypoplastic parameters (PSM). Parameters for the increase of  $r_e$  and  $M/M_c$  with  $\epsilon_a/\epsilon_c$  and the factor  $a_c$  are additionally needed. The case without isochoric trend may be called *meso-hysteresis* as it occurs at the scale of samples.

Evolutions of the average stress with these attractors as asymptotes can be captured by

$$\dot{\sigma}_i = M_{ij}[\dot{\epsilon}_j - (-a\bar{p}/p_e + b\epsilon_a + c\epsilon_c^m)m_j] \quad (5)$$

wherein the square instead of a dot denotes rates per number of cycles instead of time. Stiffness  $M_{ij}$  and flow direction  $m_j$  depend on  $\sigma_i - p_d$  instead of  $\sigma_i$  by hypoplastic relations (Niemunis 2003, PSM). Therein the seismic *pulsation pressure* is  $p_d = \bar{p}(1 - M/M_c)$  so that a steady state is obtained for  $\epsilon_a/\epsilon_c > 0$  as shown in Fig. 2b. The factors  $a, b$  and  $c$  depend on  $\epsilon_a/\epsilon_c$  so that Fig. 2d and e are reproduced.

Further specification and calibration require experiments and numerical simulations. Evolutions with stress cycles can be incorporated by means of control cycles with an elastoplastic or hypoplastic relation with hidden variables. This idea is taken over from Niemunis et al. (2005) and enables to use stress-controlled tests. (5) resembles the relation by Niemunis et al. (2005), but requires less parameters and implies the attractors. The extension with tensors for arbitrary deformations is likewise straightforward, this is needed for frame-indifference and applications to boundary value problems. The attractors enable a calibration by a single multi-stage triaxial test.

(5) is *energetically justified* by means of (3). Rate-independence is obtained with period times which exceed seismic half-lives by far (STG). Except for big  $\epsilon_a/\epsilon_c$ , where hypoplastic behavior with nearly stationary  $T_g$  is obtained, the *seismicity pulsates* between very low hypoelastic and moderate hypoplastic amounts. The term  $\dot{\alpha}\pi_i$  turned out negligible in simulations with lab-typical  $D$ , this is obtained with the proposed  $\alpha(T_g)$  (STG). The integration of  $\dot{\sigma}_i$  over one period, for getting average changes of shape and state in one cycle by division through its duration, can be simplified by taking averages of stiffness  $H_{ij}$  and elastic strain  $u_i$ . The stronger pulsation of  $\lambda\sqrt{\eta_g/\gamma}$  between ca. 1 and 100, and of  $\alpha$  between 0 and ca. 0.8, is nearly cancelled in the term  $(1 - \alpha)^2\dot{\epsilon}_i$  so that the average trend remains as the first term in (5). The pulsation of  $(1 - \alpha)\lambda\sqrt{\eta_g/\gamma}$ , however, leads to additional terms as in (5) and causes an average reduction of  $u_i$  which is represented by  $p_d$ . The average differential stiffness  $M_{ij}$  is smaller than the elastic  $H_{ij}$  due to the average transmission by  $(1 - \alpha)$ .

This qualitative support of the proposed *seismo-hypoplastic* relation will not be impaired by improvements of GSH. Qualitatively realistic attractors are obtained numerically with the present version (STG). The pressure  $p_d$  represents the average softening by the pulsating seismicity, it works like an excess pore pressure. The nearly complete seismic relaxation for  $\epsilon_a = 0$  and small  $\epsilon_c$  by (5) comes close to a seismodynamic equilibrium, the pressure  $\bar{p}\alpha\epsilon_c^m$  with  $m \approx 2$  is due to average jamming in case of sufficient density. The stronger jamming with increasing  $\epsilon_a/\epsilon_c$  is due to the built-up of elastic strain by the first term in (2). The hypoplastic limit by (5) for big  $\epsilon_a/\epsilon_c$  will also be obtained with variants of GSH.

## 4 STABILIZATION AND DESTABILIZATION

A model tower stands safely upon an *elastic base* with a wide foundation plate, Fig. 3a. A laser pointer on top shows at the ceiling that the tower returns to an upright position (autogeneous attractor) after any small disturbance. The free energy of this conservative system,  $F = F_g + F_e$  with gravitational and elastic parts, changes with a tilt  $\psi$  from  $F_0$  for  $\psi = 0$  by  $(-a_g + a_e)\psi^2$ . Therein  $a_g$  is given by mass and centre of gravity,  $a_e$  by the rotational base stiffness. Stability requires  $a_g < a_e$  so that  $F$  is minimal for  $\psi = 0$ .

With a smaller plate the tower is no more stable (Fig. 3b) as it has  $a_g > a_e$ . Then the upright position is an autogeneous *repeller* as any tilt causes a spontaneous increase of kinetic energy at the expense of  $F_g$ . At a critical point with  $a_g = a_e$  the upright position is indifferent as  $F$  does not change with small  $\psi$ . More precisely speaking, there is a fan of eigendirections with vanishing second variation of the free energy  $F$ . This holds true also for conservative systems with more degrees of freedom: equilibrium means  $\delta F = 0$ , at a critical point a collapse mode is given by an eigenvector for  $\delta^2 F = 0$ , beyond it kinetic energy is released.

The tower with the wider plate stands also safely upon dense *dry sand*. If it is quasi-statically loaded by a swing (Fig. 3c) it remains in an upright position, but sinks gradually by minute amounts. If it was slightly tilted after placement on dense sand it returns gradually by swinging (one can see that with the laser pointer) and

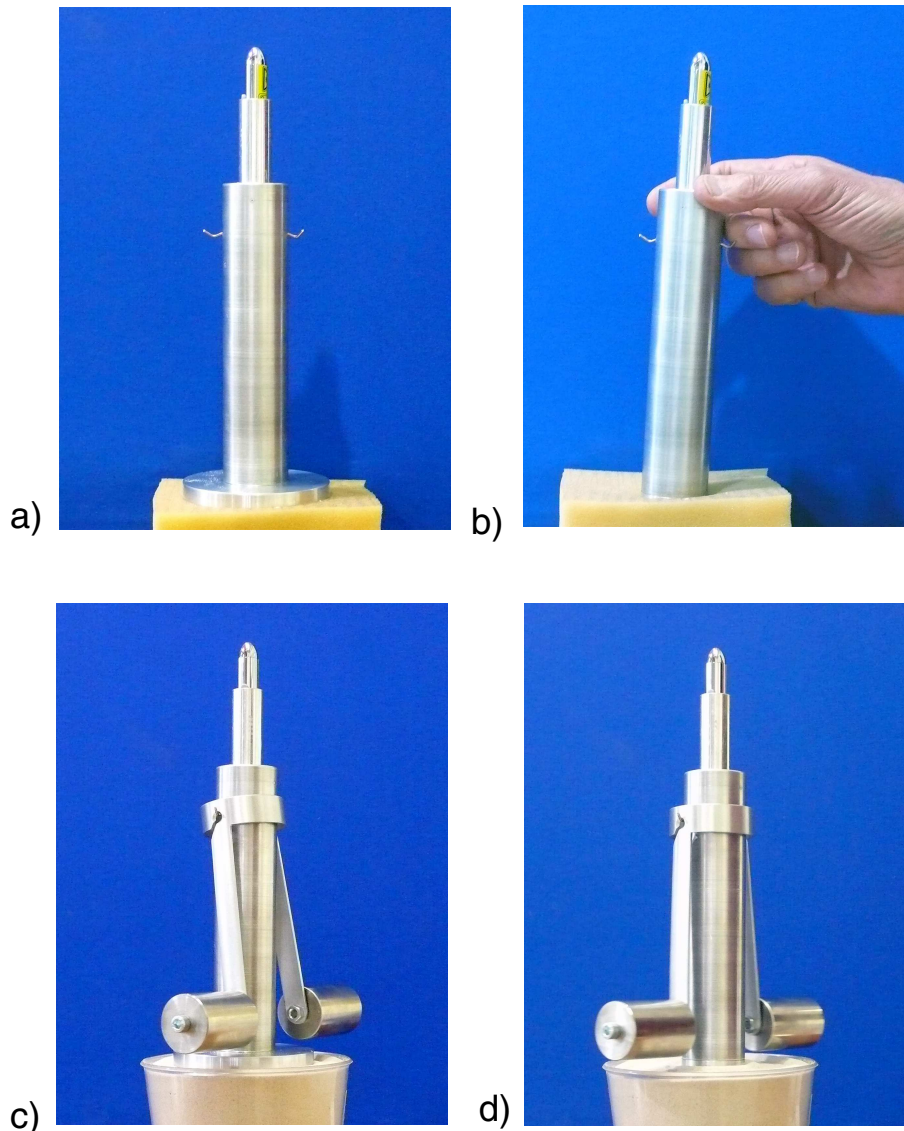


Figure 3: Model tower upon elastic base with sufficient (a) and insufficient foundation (b), and upon dense sand with stabilization (c) and destabilization (d) by a swing

goes on to sink slightly. A microphone reveals a pulsating crackling noise, so one may speak of a seismogeneous attractor with *macro-hysteresis*. This *stabilization* occurs also by repeated shocks at the base like an earthquake.

The tower with the smaller plate on dense sand (Fig. 3d) behaves differently. It tilts and sinks gradually from an upright position by swinging, so we have a seismogeneous repeller. When the tower has reached a critical tilt it topples spontaneously with more and more crackling noise. This seismogeneous *destabilization* occurs likewise with repeated base shocks. The collapse requires an amplitude-independent critical tilt and has a mode which is determined by mass and dimensions of the tower plus foundation and by the density of the sand nearby.

A stabilization occurred repeatedly with a TV tower in Moscow and with gravity offshore structures: after tilting in a gale the structures turned upright again and sunk gradually with normal wind or waves. The seismogeneous stabilization with re-densification can occur without back-tilt, e.g. with piles, and the gradual displacements can impair the serviceability, e.g. of machines or tracks. A destabilization can happen more dramatically with water: structures can suddenly slump in loose saturated sand, originally dense sand can be gradually dilated and offshore structures can topple after having reached a critical point.

The importance is evident, but published mechanical models are rather insufficient. So can the proposed

psammodynamic concept do better, at least for our model towers on dry sand? The free energy is again  $F = F_g + F_e$  as its seismic part is negligible (cf. Sec. 2), but its degrees of freedom are not as simple and unique as for conservative systems with elastic ground. The pulsating seismicity triggers rearrangements and stress redistributions,  $F$  drops gradually with decreasing rate for stabilization and increasing rate for destabilization. At a critical point  $F$  drops spontaneously with increasing seismicity and kinetic energy of the average motion, until the toppling tower reaches the ground.

The swing produces quasi-static evolutions in the sand, like wind or waves *in situ*. The specific seismic energy  $w_e$ , though far smaller than the elastic one  $w_e$  except near free surfaces, can be bigger than the kinetic energy of average displacements as long as the system is stable. Then the evolution is rate-independent as shown in Sec. 3, a tensorial extension of (5) could be employed. The field of cyclic deformation amplitudes could be calculated with a validated and calibrated elastoplastic or hypoplastic model with hidden variables for a few control cycles, as proposed by Niemunis et al. (2005) an updating is needed after substantial changes of average position and state field. This works also with constraints by pore water and embedded elastic structures, then the relaxation of the grain skeleton by pulsating seismicity can play a bigger role.

This approach works also with destabilizations, but was not yet properly worked out for critical points. For a tower upon dry sand an impending collapse may be indicated by the impossibility to get quasi-static hypoplastic solutions. A proper mode of collapse should result from an eigenvalue of the total free energy as a functional of configuration and state field. It is not yet known how this could work with finite elements, i.e. with an eigenvector in a Sobolev space. It appears that spontaneous granular chain reactions get increasingly fractal by localizations, so extended continuum approaches are required (with fractional calculus?). Localizations are more complex with pore water and gas if the grain skeleton opens into channels or films (PSM).

## 5 CONCLUSIONS AND OUTLOOK

The dynamics of sand-like matter (psammoids), called psammodynamics, can be captured by means of attractors and energetics. Monotonous deformations cause an alignment of the internal state so that this is not needed explicitly. The response is better captured by hypoplastic than by elastoplastic relations, this is justified with a granular temperature which is proportional to the intensity of stretching  $D$ . The seismic energy is stationary for constant  $D$  as its loss into heat is compensated by its generation by  $D$ . At state limits the elastic energy is at the verge of convexity, thus the seismic energy grows dramatically by imposed work and triggers localizations.

Samples with average plus cyclic deformations tend to stress cycles or skeleton decay. These attractors are partly reproduced by elastoplastic or hypoplastic relations with back stress or intergranular strain, respectively, as hidden variables. Attractors are proposed for evolutions of average shape and state, and also obtained with a novel seismo-hypoplastic relation. This is justified with a pulsating seismicity in the subcritical range which causes an average softening of the grain skeleton. As for monotonous deformations the rate-independence is justified as  $D$  is nearly constant over times which exceed by far the time for the transition of seismic energy into heat. The crackling noise and the range of hysteresis from micro (grain) to meso (sample) indicate fractality.

Boundary conditions with pulsating seismicity (macro-hysteresis) can lead to stabilization or destabilization of towers upon sand. Depending on the foundation width the upright position can work as attractor or repeller. In the second case the tower tends to a critical tilt where it topples by itself. This concept can be transferred to other cases including pore water and elastic solids, but the loss of stability is not yet well understood. The mode of a collapse is principally determined by an eigenvector of the free energy at the verge of convexity. The subsequent granular chain reaction as a succession of eigenvalue problems is more complex by increasing fractality.

The seismodynamic approach will be improved with refined energies, particularly for the verge of convexity. For a while elastoplastic, hypoplastic and seismo-hypoplastic models will serve to the purpose as far as they are validated and calibrated. The approach could also be applied to *solids*. Their acoustic emission in the ductile (stable) regime indicates seismicity from micropores. Its intensity or *seismocrasy* (analog of Greek thermocrasia = temperature)  $T_s$  enhances relaxation and dissipation so that stresses evolve with strain similarly as by (3). This could support and improve elasto- and hypoplastic relations (PSM).

The indirectly audible seismic kinks may be called *seismons* as they resemble phonons in solids and remind of radioactivity observed with a Geiger counter. The similarity to quantum mechanics may be more than accidental, but we are far from a seismic counterpart of Schrödinger's equation. The *thermal activation* by phonons comes into play with very slow evolutions and/or softer solid particles. Quartz sand under tectonic

deformation with  $D \approx 10^{-13}/s$ , e.g., has viscous creep-relaxation as phonons matter more than seismons. Clays with lower activation energies feel seismons and phonons likewise for geotechnically usual  $D$ . An extension of (2) for thermal relaxation could support and improve viscoplastic relations (PSM).

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