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SAND ANCHORS — A SHEAR ZONE PROBLEM

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ABSTRACT

Sand anchors represent a new progressive technology for soil and rock stabilization. They consist of a steel rod embedded in dense sand. The wall of the anchor borehole acts as a constraint for the sand dilatancy and its stiffness has a decisive influence on the pull-out force. Due to their high stiffness, rocks can be considered as an optimum milieu for the application of sand anchors. The force-displacement behaviour is influenced mostly by sand density, grain diameter and roughness of both the anchor rod and the borehole wall.

Numerical calculations of sand anchors have been performed using a hypoplastic constitutive relation. The constitutive parameters are closely related to index properties of sand. The model takes into account pressure and density effects. A description of the deformation mechanism by a single element test can already yield realistic results. A more detailed picture has been obtained with FE-calculations using a polar extension of the hypoplastic relation within a Cosserat continuum. In this manner, the influence of the grain size and the thickness of shear zones could be determined. For the maximum pull-out force, the optimal shear zone extends over the whole space between the wall and the anchor rod; this requires a suitable grain size. A comparison with model tests has confirmed the numerical results.

KEYWORDS

Hypoplasticity, anchor, sand, limit load, Cosserat continuum, shear zone, FE-calculation

INTRODUCTION

Rock anchors consisting of anchor rods embedded in dry mineral granulates generally offer several advantages in comparison to standard grouted anchors:
- the method is reversible, for example the stabilising devices can be exchanged or removed;
- the load is applicable immediately after construction, and drainage of the soil around the anchor is provided;
- due to the short installation time, the use of relatively inexpensive materials and small machinery, costs are kept to a minimum.

The layout and technology to install sand anchors and the mechanisms which are involved in the static bearing capacity are explained, and the most important parameters influencing the design of sand anchors are discussed. A hypoplastic constitutive law used for an analysis is shorly outlined followed by calculations of element tests which can be used for an estimation of forces and displacements in a special case. Finally, results of FEM-calculations with a polar hypoplastic approach for simple shear and a sand anchor are shown.

MODEL TESTS

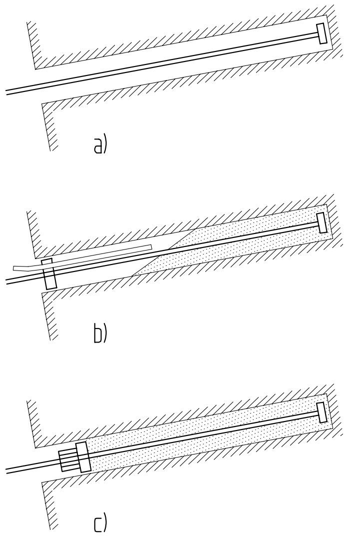

Figure 1: Principle of construction of a granular anchor in rock

The installation of a granular anchor is shown in three sketches in Figure 1:
a) A hole is drilled in the rock and a steel or a glas fiber rod is inserted with an end plate.
b) A granulate is blown into the borehole through a tube using air pressure, filling the borehole from the deepest point to the stopper.
c) The head of the anchor is attached to the rod by means of a nut, and then the anchor is prestressed.

Figure 2 shows an ideal granular material with different densities. Consider a material which is subjected to shearing at maximum density. In this case it tends to increase its volume up to the critical density. But if we consider a sand anchor in rock, the dilatancy of sand is constrained, and therefore high shear forces can develop until the grains are destroyed. For sand anchors in weak rock the bearing capacity is not that high, because the borehole walls are deforming and the dilatancy is only partly constrained.

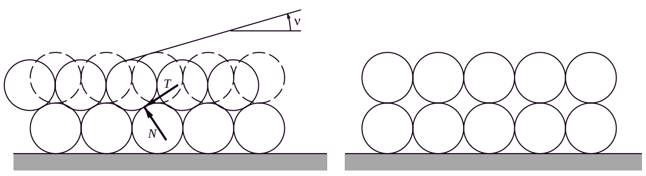

Figure 2: Maximum and critical density of an ideal granular material

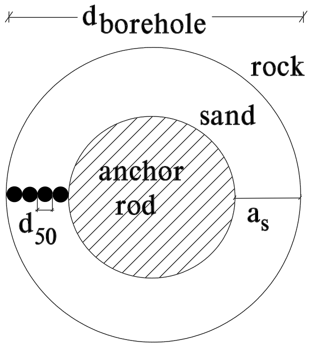

Figure 3: Cross-section

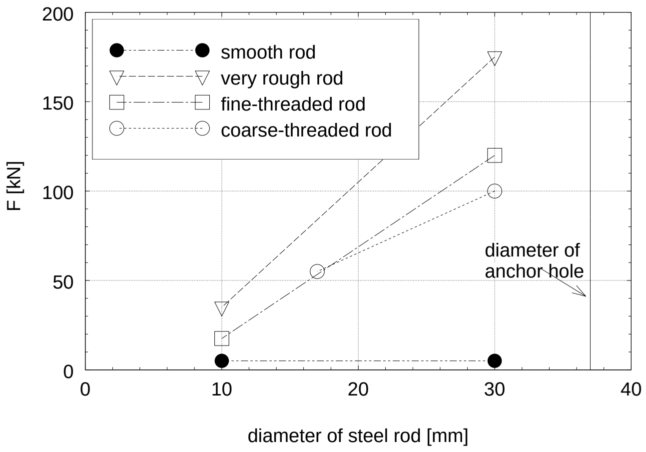

Figure 4: Pull-out force vs roughness of anchor rod

Vertical pull-out tests with sand anchors in sandstone have been performed (Wehr, 1997a,b,c). The cross-section of the sand anchor is shown (Figure 3) with a diameter of the borehole of 37 mm which has been kept constant for all the tests.

The roughness of the anchor rod has the greatest influence on the pull-out force (Figure 4; other parameters, except of the diameter of the steel rod, are constant here). Rods with four different roughnesses have been used: smooth, where the pull-out force is nearly zero, a coarse-threaded rod and a fine-threaded rod, where the force is larger, and a very rough rod with sand glued to the steel surface. In the

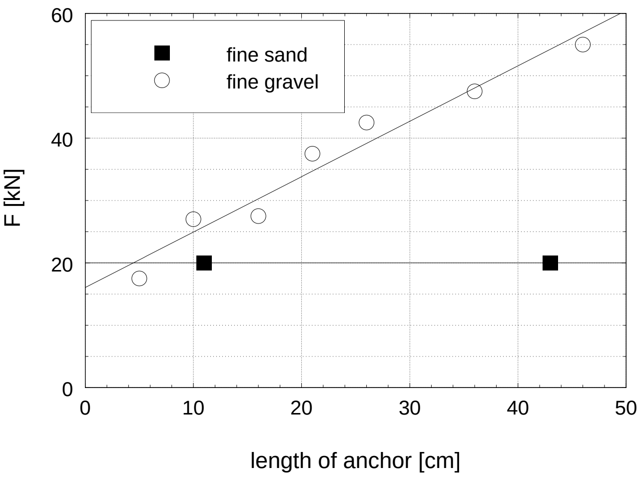

Figure 5: Pull-out force vs anchor length

last case forces up to 180 kN have been measured with a length of the rod of 40 cm. The pull-out force of such a sand anchor is limited only by the tensile strength of the rod.

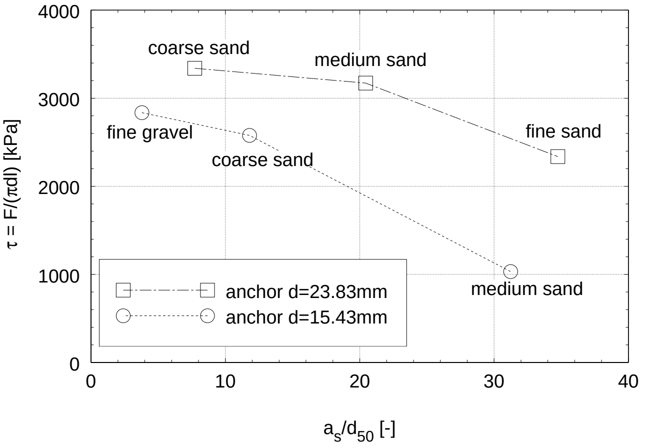

Figure 6: Pull-out shear force along the shaft vs ratio of the width of the annular space to the mean grain diameter of sand

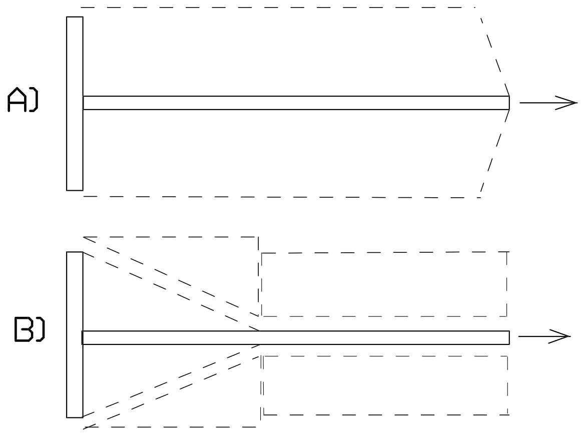

Figure 7: 'Global' A) and 'local' B) failure mechanism of sand anchors with an end plate

In Figure 5, using a constant diameter of the rod and borehole, a linear increase of the pull-out force with length can be observed with fine gravel, but no increase is obtained with fine sand. This effect depends on two important ratios: r_r/d_{50}, the roughness of the steel rod to the mean grain size, and a_s/d_{50}, the thickness of the annular space between the rod and the wall to the mean grain size (Figure 3). Because the roughness of the steel rod is very large in comparison to the mean grain size in both cases, the second ratio dominates here. There are two different failure mechnisms (Figure 7): for a fine gravel we have a 'global' failure, which means that the dilatancy constraint occurs along the total length of the rod, and only one shear zone develops across the total width between rod and wall. In contrast, for a fine sand the shear zone along the rod is not able to spread over the whole width, because there are too many grains in the annular space. An additional cone-shaped shear zone develops near the end plate which may be called a 'local' failure. This second mechanism was modelled with a simple equation by Stazhevsky *et al.* (1995).

The effect of the a_s/d_{50} ratio is shown in Figure 6. Rough rods with two different diameters were subjected to a pull-out force using three different mean grain diameters. The pull-out shear force along the shaft shows a larger value for the larger anchor diameter. Independently of the anchor diameter, the maximum shear force is reached for a_s between 4 and $6{\cdot}d_{50}$; it decreases slightly up to $20{\cdot}d_{50}$, then it falls down rapidly and reaches a constant value for $a_s > 30{\cdot}d_{50}$.

Summarising, we observe a small decrease of the shear resistance with increasing thickness of the shear zone, and a large decrease if the shear zone becomes smaller than the width of the annular space. Thus, high pull-out forces can only been observed for an annular space smaller than the thickness of the shear zone.

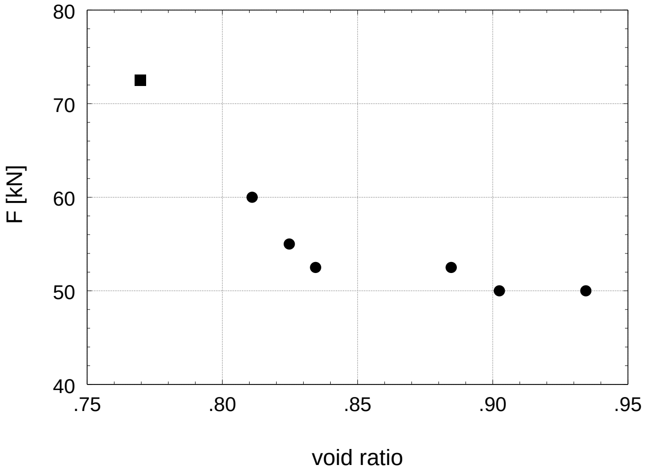

Figure 8: Pull-out force vs void ratio ($e_{min} = 0.6$, $e_{max} = 1.0$)

Other parameters influencing the pull-out force are: the roughness of the borehole wall, the void ratio (Figure 8), the quartz content and the roundness of grains.

Dynamic tests with sand anchors in rock are described by Wehr (1997b).

HYPOPLASTIC MODEL

The nonpolar hypoplastic model used here takes into account the effects of mean pressure and density and is described in detail by Gudehus (1996) and Bauer

(1996). It is represented by a tensorial equation which yields a co-rotated (Jaumann) stress rate $\overset{\circ}{\mathbf{T}}$ as a function of the stress $\mathbf{T}$, deformation rate $\mathbf{D}$ and void ratio e. Eight material parameters are needed to characterize a granular material. The critical friction angle φ_c can be obtained as the angle of repose; e_{c0} and e_{d0} are the critical and minimum (through cyclic shearing) void ratios at zero pressure and they correspond approximately to e_{max} and e_{min} from standard tests; the maximum void ratio in an isotropic state at zero pressure e_{i0} can be estimated as $1.2e_{c0}$; the granulate hardness h_s and the exponent n can be calculated from the oedometric compression curve with a loose specimen which can be approximated by

$$e = e_0 \ \exp[-(3p_s/h_s)^n] \tag{1}$$

(e_0 is the void ratio at the mean pressure $p_s = 0$); and the exponents α and β can be determined from the peak friction angle and the compression coefficient, respectively, for a dense specimen (Herle, 1997).

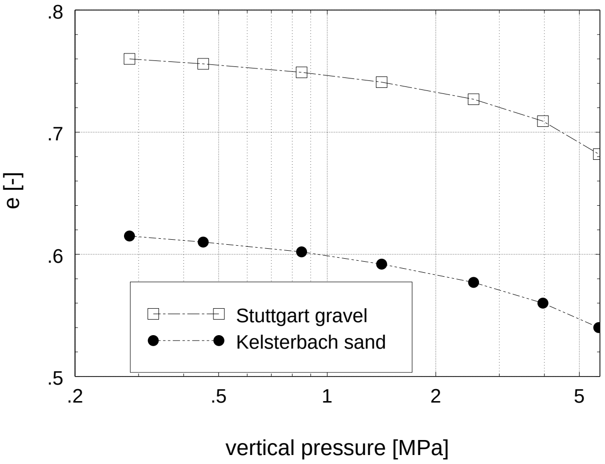

Figure 9: Oedometric compression curves of Stuttgart gravel and Kelsterbach sand.

Two oedometric compression curves used for the determination of h_s and n are depicted in Figure 9. An upper bound pressure corresponds to the maximum pressure level in the model tests, $\sigma_{max} = \tau/\tan\varphi_p = 5$ MPa, using an estimated peak friction angle $\varphi_p = 35°$ (see Figure 6). The parameter n reflects the curvature of the compression curve and is calculated as

$$n = \ln[(\lambda_2 e_1)/(\lambda_1 e_2)]/\ln(p_{s2}/p_{s1}) \ , \tag{2}$$

λ_i and e_i correspond to the compression coefficients and void ratios, respectively, at the minimum and maximum pressures p_{si}, i=1,2. The granulate hardness

$$h_s = 3p_s(ne/\lambda)^{1/n} \tag{3}$$

takes into account the slope of the compression curve (Herle, 1997).

In Table 1, the parameters of the hypoplastic model for two granular materials considered in recalculations of model tests are summarized.

TABLE 1: MATERIAL PARAMETERS FOR THE HYPOPLASTIC MODEL

material	φ_c [°]	h_s [MPa]	n [–]	e_{d0} [–]	e_{c0} [–]	e_{i0} [–]	α [–]	β [–]
Stuttgart gravel	37	190	0.52	0.58	0.88	1.06	0.25	1.5
Kelsterbach sand	33	290	0.42	0.52	0.82	1.00	0.25	1.1

An extension of hypoplasticity for a polar Cosserat continuum following Tejchman (1997) was also used. Herein, the characteristic length corresponds to the mean grain diameter d_{50}. The tensor of the deformation rate $\mathbf{D}_c$ is non-symmetric, and an additional degree of freedom, rotation ω^c, is connected with the vector of couple stresses m through the curvature vector $\mathbf{k} = \partial\omega^c/\partial\mathbf{x}$ for plane strain and axially symmetric cases. The additional parameter $a_m = 1$ was obtained from a comparison of numerical and analytical solutions of simple shearing of an infinite sand layer between two very rough boundaries (Bauer and Tejchman, 1995).

CALCULATION OF SIMPLE SHEAR TESTS

Non-polar calculation

Let us consider two model tests in rock with constrained dilatancy. The first test with an anchor diameter of $d = 15.4$ mm and fine gravel, and the second test with $d = 23.8$ mm and coarse sand. In both cases the width of the annular space between anchor rod and wall corresponds to $4d_{50}$. Approximately, this deformation mechanism can be regarded as simple shearing with constant volume because the radial displacements are fully constrained. It can be modelled by a numerical element test because the deformation of the sand is approximately homogeneous due to the width of the shear zone over the whole annular space.

Figure 10 represents force-displacement curves of the anchor in the model tests and calculations. The chosen void ratios of both soils correspond to a rela-

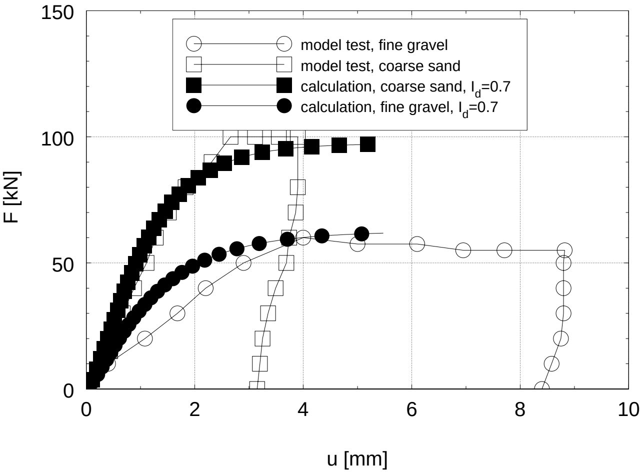

Figure 10: Comparison of the calculated constant volume simple shearing with results of model tests

tive density I_d = 0.7 which was achieved in experiments through a special preparation method.

The calculated curves are close to those from the model tests, indicating that the assumption of a homogeneous displacement in the shear zone between the anchor rod and the borehole wall was reasonable. In this case, the calculation of an element test with a nonpolar continuum can yield an estimation of the pull-out force.

Polar calculation

A simple shear test of an infinite sand layer between two walls have been modeled with FEM (Bauer and Tejchman, 1995, Tejchman, 1997). A polar approach for the planar sand layer with a height of 2 cm and a mean diameter of the grains of $d_{50} = 0.5$ mm have been used. The sand was considered between two very rough walls under full constraint of dilatancy, and therefore the shear stresses inside the sand layer increased up to 3.5 MPa and the normal pressures up to 6 MPa after a displacement of the rod of 2 cm.

The polar effect was significant in the shear zone, i.e. Cosserat rotations and couple stresses were noticeable. The grain diameter, wall roughness, stiffness of the wall and the height of the sand layer have been varied in the calculations. The smaller the grain diameter, the wall roughness, the stiffness of the wall and the larger the height of the sand layer, the smaller is the thickness of the shear zone, which was calculated as $8d_{50}$ for a rough wall and $4d_{50}$ for a smooth wall.

If d_{50}, the wall roughness and stiffness are too small or the layer height is too large, the thickness of the shear zone, clearly demonstrated by Cosserat rotations, can become smaller than the width of the entire layer. Evidently, the effect of the mean grain diameter can only be modeled within a polar continuum.

FE-CALCULATIONS

The FEM-analysis of granular anchors in rock was modeled with a polar hypoplastic constitutive law under plane strain conditions. The considered dimensions of the sand body were 10 × 1 cm (height × width). They correspond to the anchor diameter of 15.4 mm of the model tests in rock.

In total, 800 three-noded triangular elements with three degrees of freedom in each node and with linear shape functions were adopted. The dimensions of elements were smaller than $5d_{50}$. The integration was performed with one sampling point in the middle of each element. The calculations were carried out with large deformations and curvatures using an updated Lagrange formulation together with the Jaumann stress rate and couple stress rate. In this way, changes of the element configuration and volume were taken into account. A quasi-static deformation was initiated through constant vertical displacement increments $\triangle u$ prescribed at the nodes along a very rough anchor rod. For the solution of the nonlinear equation system, a modified Newton-Raphson scheme was used with an initial global stiffness matrix, and for the time integration an explicit Euler forward scheme was used. Tension stresses in sand were not allowed.

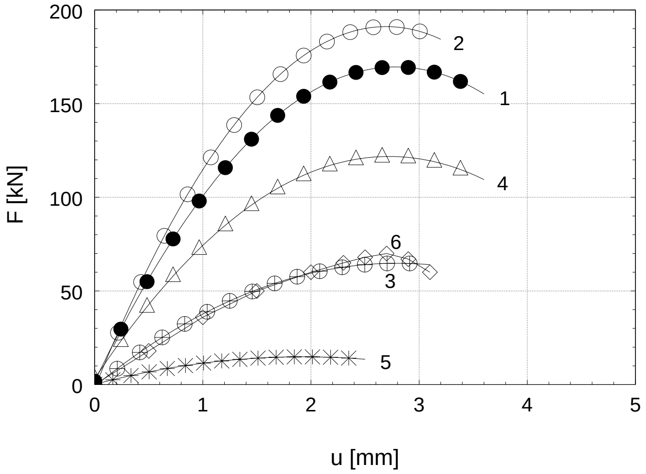

Figure 11: Calculated force vs displacement

Figure 11 shows the calculated anchor force against the vertical displacement of the anchor rod. The different curves may be compared using Table 2. The anchor force increases with decreasing initial void ratio of the sand (curves 3,1) and with increasing mean grain size (curves 1,2), wall roughness (curves 6,1) and stiffness of the borehole wall (curves 5,4,1) modelled by means of horizontal springs.

TABLE 2: ANNOTATION OF CURVES IN FIGURE 11

curve	e_0 [-]	d_{50} [mm]	r_w [mm]	k [kN/m]
1	0.60	0.50	d_{50}	∞
2	0.60	0.75	d_{50}	∞
3	0.65	0.50	d_{50}	∞
4	0.60	0.50	d_{50}	10000
5	0.60	0.50	d_{50}	100
6	0.60	0.50	$d_{50}/10$	∞

e_0 = initial void ratio, d_{50} = mean grain diameter, r_w = wall roughness, k = stiffness of elastic springs of the wall

The FE-calculations of sand anchors in rock yield qualitatively and quantitatively realistic results. The important effects which have been observed in the model tests were described with polar hypoplastic FE-calculations.

CONCLUSIONS

Granular anchors are a good alternative to traditional methods. The same bearing capacity as for grouted anchors and nails can be reached.

The advantages are short time of installation, application of the load immediately afterwards, reversibility of temporary anchors, and lower costs.

Model tests of sand anchors have been carried out in intact sandstone. The most important parameters are the roughness of the anchor rod and borehole wall, and the length and void ratio of the granular body. The roughness of the anchor rod and the borehole wall should be large with respect to d_{50} and the width of the annular space between the rod and the wall should be small (ca. $4d_{50}$). The pull-out force grows with increasing sand density, mean grain diameter, rod roughness and borehole stiffness and with decreasing annular space between the rod and the wall.

Numerical calculations of sand anchors have been performed using a hypoplastic constitutive relation. A description of the deformation mechanism by a single element test can already yield realistic results. A more detailed picture can be obtained with FE-calculations using a polar extension of the hypoplastic relation within a Cosserat continuum. In this way, the influence of the grain size and the thickness of shear zones can be determined. The maximum pull-out force can be reached if the shear zone extends over the whole space between the wall and the anchor rod; this requires a suitable grain size. A comparison with model tests has confirmed the numerical results.

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